Numerical Computation of the Interaction Between Electromagnetic Waves and Nonlinear Superconducting Materials

Salvatore Caorsi, *Member, IEEE*, Andrea Massa, and Matteo Pastorino, *Senior Member, IEEE*

***Abstract*—In this paper, we propose a numerical solution to the computation of the interaction between electromagnetic waves and superconducting materials. A superconductor is identified as a material with a negative dielectric permittivity on the basis of the two-fluid model and an integral-equation formulation is developed. The approach takes into account the nonlinear behavior (i.e., the dependence of the material parameters on the internal magnetic field), experimentally observed in previous studies. An iterative process is then developed for the numerical solution; the process is based on the distorted-wave Born approximation. The mathematical formulation of the approach is described and some numerical results concerning the canonical case of a circular cylinder and the behavior of two superconducting transmission lines are reported.**

***Index Terms*—Microwave, scattering, superconductivity, transmission lines.**

I. Introduction

IN THE PAST few years, there has been a growing interest in the applications of superconducting materials in a large number of electronics and telecommunications areas [1]–[4], especially in the light of the recent discovery of high-T_c superconductors. For the microwave community, the application of superconducting materials appears to be of great importance, e.g., in the design of microwave circuits [5] and antennas [6] and in the development of a variety of devices based on planar transmission lines, which have been intensively studied both theoretically and experimentally [7]–[16].

This wide interest in microwave superconductivity requires that considerable effort be devoted to a better understanding of the physical nature of such materials on the basis of a suitable microscopic theory. On the other hand, it is necessary that, starting from a sufficiently justified macroscopic model, rigorous methods be devised to accurately predict the performances of developed devices. In the final analysis, this requires the study of the electromagnetic behavior of superconducting materials, e.g., the description of the interaction between electromagnetic waves and superconductors.

In this area, an important role is played by computational electromagnetics methods, on which a very comprehensive paper was recently published [17]. In [17], Mei and Liang provided an understanding of superconducting materials, developed in the context of classical electrodynamics. They stated that the solutions of electromagnetic boundary-value problems are not any more complicated for a superconductor than for any other penetrable medium, once it is treated as a negative dielectric material.

This paper is essentially a computational one. It is aimed at the study of the interaction between electromagnetic waves and superconducting materials, starting from the two-fluid model and a macroscopic description in terms of the Maxwell and London equations.

Recently, efforts were also concentrated on taking into account the significant nonlinear response of superconducting materials. Such a response was experimentally observed and deeply studied in several papers (see, e.g., [18]–[26]). In particular, the dependence of the surface resistance R_s and of the penetration depth λ on the microwave magnetic field has been pointed out. This paper is aimed at taking into account these nonlinear effects, in particular, by considering those cases for which the nonlinearity was found to be well approximated by a quadratic nonlinearity, that is analogous, in a sense, to the Kerr-like nonlinearity [27], widely assumed for nonlinear dielectrics [28]. In recent years, we developed several computational techniques for the study of the electromagnetic behavior of nonlinear dielectric materials. They were numerical techniques, either “exact” [29] (i.e., without approximations different from numerical ones) or iterative [30], the latter being based on approximations like the distorted-wave Born approximation and the Rytov approximation.

In the following, the assumed model of a superconducting material will be outlined. The description of the computational procedure used for the study of the interaction between electromagnetic waves and superconducting materials, taking into account nonlinear effects, will then be provided. Finally, some results will be reported.

II. Theory

A. Complex Permittivity

In the two-fluid model, the total current density flowing in a superconductor is the sum of two terms related to normal-particle and superparticle charge densities

$$\mathbf{J} = \mathbf{J}_n + \mathbf{J}_s. \tag{1}$$

Manuscript received November 10, 1999; revised December 1, 2000.

S. Caorsi is with the Department of Electronics, University of Pavia, I-27100 Pavia, Italy (e-mail: caorsi@dibe.unige.it).

A. Massa is with the Department of Civil and Environmental Engineering, University of Trento, I-38050 Trento, Italy (e-mail: andrea.massa@ing.unitn.it).

M. Pastorino is with the Department of Biophysical and Electronic Engineering, University of Genoa, I-16145 Genova, Italy (e-mail: pastorino@dibe.unige.it).

Publisher Item Identifier S 0018-9480(01)08671-9.

0018–9480/01$10.00 © 2001 IEEE

The two components of $\mathbf{J}$ are related to the field by Ohm's law and by the London equation

$$\mathbf{J}_n = \sigma_1(T)\mathbf{E} \quad (2)$$

$$\mathbf{J}_s = \frac{\mathbf{E}}{j\omega\Lambda(T)} \quad (3)$$

where $\sigma_1(T)$ is the electric conductivity given by $\sigma_1(T) = \sigma_n(T/T_c)^4$ (where σ_n is the normal-state conductivity), T is the temperature, and T_c is the critical temperature. In (3), ω is the frequency and $\Lambda(T)$ is given by

$$\Lambda(T) = \mu_0\lambda^2(T) \quad (4)$$

where μ_0 is the magnetic permeability of vacuum and λ is the penetration depth of the superconductor.

By using the Maxwell equation

$$\nabla \times \mathbf{H} = j\omega\varepsilon_0\mathbf{E} + \mathbf{J}_n + \mathbf{J}_s = j\omega\varepsilon_0\varepsilon_r^*\mathbf{E} \quad (5)$$

one can define an effective complex dielectric constant

$$\varepsilon_r^* = 1 - \frac{1}{\omega^2\varepsilon_0\mu_0\lambda^2} - j\frac{\sigma_1}{\omega\varepsilon_0} = \varepsilon_r' - j\varepsilon_r'' \quad (6)$$

Since ε_r' can also be expressed as [17]

$$\varepsilon_r' = 1 - \frac{\omega_s^2}{\omega^2} \quad (7)$$

where ω_s is a critical frequency of the order of 1–10 THz for most superconductors [17], for $\omega < \omega_s$, it results $\varepsilon_r' < 0$.

B. Nonlinearity With Respect to the Magnetic Field

Nonlinearity is macroscopically modeled starting from the experimental results reported in [18], obtained by using superconducting thin films. Although more recent high-temperature superconductor (HTS) films have been found to be less nonlinear, the nonlinear behavior has been confirmed. In those experimental results, by using $YBa_2Cu_3O_{7-x}$ superconducting films, it was found that, at 4 K below the critical field and at 77 K over the entire range, the surface resistance is well approximated by the following quadratic relation [18, p. 1526]:

$$R_s = R_{s0} + \alpha H_{\rm rf}^2 \quad (8)$$

where R_{s0} is the R_s computed for a very low field and $H_{\rm rf}$ is the maximum magnetic field. In [31], these authors demonstrated that the quadratic relation (8) is consistent with the Ginsburg–Landau theory [2]. Moreover, the same authors calculated the change in penetration depth as a function of the RF magnetic field. They found that the fractional change $\Delta\lambda/\lambda$ depends in a nonlinear way on $H_{\rm rf}$. The nonlinear behavior given in [18, Fig. 9] can be also approximated as a quadratic function

$$\frac{\Delta\lambda}{\lambda} = \zeta H_{\rm rf}^2. \quad (9)$$

This relation will be used in the following. Actually, the authors found that the fractional change in λ is smaller than the change in R_s and concluded that the change in λ is not nearly large enough to explain the change in R_s. It should be noted that this is not a surprise considering that the real part of the complex conductivity is much less than the imaginary part and how R_s and λ depend on these quantities. However, the $\Delta\lambda/\lambda$ effect is amplified in high-Q resonant devices. On the basis of the above considerations, the nonlinearity of a superconducting material can be expressed through the following nonlinear relationship for the conductivity σ_1 (without considering the quadratic behavior of the fractional change in λ):

$$\sigma_1 = \sigma_{10} + \frac{2\alpha}{\omega^2\mu^2\lambda^3}H_{\rm rf}^2 = \sigma_{10} + \beta H_{\rm rf}^2 \quad (10)$$

with $\beta = (2\alpha)/(\omega^2\mu^2\lambda^3)$. This nonlinearity has a quadratic form similar to the Kerr-like nonlinearity for dielectrics, in which the (positive) dielectric permittivity turns out to depend on the square of the internal electric field [36]. By using (10), (6) can be rewritten as

$$\varepsilon_r^* = \varepsilon_{r_{\rm L}} + \varepsilon_{r_{\rm NL}} \quad (11)$$

where $\varepsilon_{r_{\rm L}} = \varepsilon_r' - j(\sigma_{10})/(\omega\varepsilon_0)$ and $\varepsilon_{r_{\rm NL}} = -j(\beta H_{\rm rf}^2)/(\omega\varepsilon_0)$.

If the fractional change in λ is taken into account, from (9) we obtain (since $\Delta\lambda/\lambda$ is small)

$$\sigma_1 \approx \sigma_{10} + \frac{2\alpha}{\omega^2\mu^2\left[\lambda_0(1+\zeta H_{\rm rf}^2)\right]^3}H_{\rm rf}^2 \approx \sigma_{10} + \beta' H_{\rm rf}^2 + \chi H_{\rm rf}^4. \quad (12)$$

where $\beta = (2\alpha)/(\omega^2\mu^2\lambda_0^3)$ and $\chi = (-6\alpha\varsigma)/(\omega^2\mu^2\lambda_0^3)$, being λ_0 given by $\lambda_0 = \lambda(H_{\rm rf} = 0)$. Considering the nonlinear behavior of λ implies to add a fourth-order term and the nonlinearity assumes a form similar to that considered, for example, in [32], and concerning the guided propagation in nonlinear dielectric media. A fourth-order nonlinearity has been already assumed by the authors for scattering computation in the case of nonlinear dielectrics [33].

Finally, according to (12), the ε_r^* is still given by (11), but $\varepsilon_{r_{\rm L}}$ is computed on the basis of λ_0, and $\varepsilon_{r_{\rm NL}}$ is now a complex quantity given by $\varepsilon_{r_{\rm NL}} = (2\zeta H_{\rm rf}^2)/(\omega\varepsilon_0\mu\lambda_0^2) - j(\beta H_{\rm rf}^2 + \zeta H_{\rm rf}^4)/(\omega\varepsilon_0)$. It should be mentioned that the proposed model does not take into account the hysteresis effect, which has been recently experimentally shown for HTSs [37].

C. Integral-Equation Formulation

As stated in [17], once a superconductor has been identified as a negative dielectric material, Maxwell's equations can be solved as for any other penetrable material. As an example, for superconducting strips in the linear case, an integral equation formulation was proposed in [7].

In this paper, we consider the interaction between an electromagnetic wave $\mathbf{E}_{\rm inc}(\mathbf{r})$ and a bounded superconducting material enclosed in a region V. By following the formulation provided in [29] for nonlinear dielectrics and by taking into account the square dependence on the magnetic field, an equivalent problem can be defined, which can be formulated by using the following Maxwell equations:

$$\nabla \times \mathbf{E} = -\mu_0\frac{\partial\mathbf{H}}{\partial t} \quad (13)$$

$$\nabla \times \mathbf{H} = \varepsilon_0\frac{\partial\mathbf{E}}{\partial t} + \mathbf{J}_{eq} \quad (14)$$

where $\mathbf{J}_{eq}$ is an equivalent current density given by

$$\mathbf{J}_{eq} = \varepsilon_0\frac{\partial}{\partial t}[\varepsilon_{r_{\rm L}} - 1]\mathbf{E} + \varepsilon_0\frac{\partial}{\partial t}\varepsilon_{r_{\rm NL}}\mathbf{E} = \mathbf{J}_{eq_{\rm L}} + \mathbf{J}_{eq_{\rm NL}}. \quad (15)$$

From (13) and (15), it is possible to derive the following inhomogeneous wave equation:

$$\nabla \times \nabla \times \mathbf{E} + \varepsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial^2 t} = -\mu_0 \frac{\partial \mathbf{J}_{eq}}{\partial t} \tag{16}$$

where the right-hand side is the exciting term dependent on the nonlinear process. It can be noticed that the linear part of the equivalent current density in relation (15) is coherent with that used in [7, eq. (12)]. Following the approach described in [29], under the hypothesis of a periodic time-dependent incident field and assuming a weak nonlinearity, one obtains the following equation for the nth vector component of the Fourier expansion of the total electric field

$$\nabla \times \nabla \times \mathbf{E}_n - k_n^2 \mathbf{E}_n = k_n^2 [\varepsilon_{r_{\rm L}} - 1] \mathbf{E}_n + k_n^2 \mathbf{Q}_n \tag{17}$$

where $k_n = n\omega(\varepsilon_0 \mu_0)^{1/2}$ and $\mathbf{Q}_n$ is given by

$$\mathbf{Q}_n = \sum_{p=-\infty}^{+\infty} \sum_{p=-\infty}^{+\infty} \gamma_{pq}^n O_p \mathbf{E}_q \tag{18}$$

where O_p denotes the nth (scalar) term of the Fourier expansion of $\varepsilon_{r_{\rm NL}}\{\mathbf{H}\}$, $\gamma_{pq}^n = 1$, if $p + q = n$ and $\gamma_{pq}^n = 0$ otherwise. It is worth noting that each vector $\mathbf{Q}_n$ generally depends on all the harmonic components of the electric field. If $\beta = 0$, $\mathbf{J}_{eq_{\rm NL}} = 0$, and (17) turns out to be representative of the scattering by a linear (negative) dielectric object. At this point, the total electric field inside and outside V, fulfilling Sommerfeld's radiation conditions, can be expressed as

$$\mathbf{E} = \mathbf{E}_n^{\rm inc} - k_n^2 \int_V [\varepsilon_{r_{\rm L}} - 1] \mathbf{E}_n \cdot \Gamma d\mathbf{r}' - k_n^2 \int_V \mathbf{Q}_n \cdot \Gamma d\mathbf{r}' \tag{19}$$

where Γ denotes the dyadic Green's function for free space [34]

$$\Gamma = -\frac{1}{4\pi|\mathbf{r} - \mathbf{r}'|} \left[I + k_n^{-2} \nabla\nabla \right] \exp\left(-jk_n |\mathbf{r} - \mathbf{r}'| \right) \tag{20}$$

where I is a unit tensor. Of course, if the problem exhibits homogeneities along one or two axes, the formulation reduces to a two- or one-dimensional one, respectively and proper forms of the Green's function must be used [34]. In particular, in this paper, we assume, for simplicity, a two-dimensional geometry with a transverse magnetic illumination with respect to the z-axis (TM_z)

$$\mathbf{E}^{\rm inc}(\mathbf{r}, t) = E^{z-\rm inc}(x, y, t)\mathbf{z} \tag{21}$$

where $\mathbf{z}$ is the unit vector of the z-axis. In this case, the above formulation for the nonlinear scattering problem can be reduced to a scalar one and, by analogy to [35], (19) can be rewritten as

$$\begin{aligned} E_n^z(x,y) =& E_n^{z-\rm inc}(x,y) + \frac{jk_n^2}{4} \int_S \left[\varepsilon_{r_{\rm L}}(x',y') - 1 \right] \\ &\times E_n^z(x',y') H_0^{(2)}(k_n \rho) dx' dy' \\ &+ \frac{jk_n^2}{4} \int_S \mathcal{Q}_n^z(x',y') H_0^{(2)}(k_n \rho) dx' dy' \end{aligned} \tag{22}$$

where S is the cross section of the scatterer, $\rho = \left[(x-x')^2 + (y-y')^2 \right]^{1/2}$, $H_0^{(2)}(k_n\rho)$ is the Hankel function of the second kind and the zeroth order, and $\mathcal{Q}_n^z(x',y')$ is the scalar analogous to $\mathbf{Q}_n$ in relation (18).

D. Numerical Solution

Since the nonlinearity has been assumed to be weak, in this paper, we explore the possibility of predicting the effects of a nonlinearity at the fundamental frequency ($n = 1$) [30], [36], which is the main responsible for the changes in σ_1 due to the square dependence on $H_{\rm rf}^2$ in relations (8) and (9). The generation of higher order harmonics and their effects on the field at the fundamental frequency is taken into account via the coupling term $\mathcal{Q}_i^z(x,y)$. After discretization of the continuous model represented by the integral (22), for $n = 1, 2, \ldots$, the problem solution is reduced to the solution of an algebraic system of nonlinear equations [29]. In particular, if we consider a fixed number N of harmonic components (a series truncation is performed) and after expanding $E_n^z(x,y)$ and $\mathcal{Q}_n^z(x,y)$ into the sum of M basis functions with the coefficients E_{nm}^z and $\mathcal{Q}_{nm}^z$ ($m = 1, \ldots, M$), the resulting system can be expressed as

$$\sum_{n=1}^N h_{nm}^z E_{nm}^z + \sum_{n=1}^N f_{nm}^z \mathcal{Q}_{nm}^z = \sum_{n=1}^N E_{nm}^{z-\rm inc} \tag{23}$$

where the coefficients h_{nm}^z and f_{nm}^z are obtained as in [35] by applying the Richmond formulation [38], which has been proven to be accurate for the forward scattering by a dielectric cylinder if a TM illumination is used [39]. It is worth noting that, for $\beta = 0$, the system (23) consists of linear equations and the adopted numerical solution becomes similar to that obtained in [7] for the field computation in the transverse section of a superconducting transmission line. An analogous formulation was used in [40] for the computation of the TM transmission of metallic conducting shields.

In the case of nonlinear dielectrics, two ways have been considered for the solution of (22). The first lies in reducing the solution of the resulting algebraic system to the minimization of a suitable cost function. In [41], we followed this approach and solved the defined large-scale optimization problem by applying a statistical cooling procedure. The second approach, in the case of weak nonlinearities, is based on the application of iterative solutions [30] using approximations like the distorted-wave Born approximation [42]. Although the former approach is more rigorous, it is usually very time consuming. On the other hand, the results obtained by using the iterative approach have been found to be in good agreement with those obtained by the statistical cooling procedure. Direct comparisons were made in [41]. Moreover, for circular cylinders with Kerr-like nonlinearities, some results were compared (with reasonable agreements) with those previously published in the literature and obtained by using a perturbation method [36]. In the following, the approach based on the distorted-wave Born approximation will be used.

III. Numerical Examples

The first example concerns the interaction between a plane wave and a circular cylinder, which, in the framework of computational electromagnetics, is considered a canonical problem since, in the linear case, an analytical solution is available. The incident field impinges normally at a frequency $f = 1.5$ GHz and the cylinder is a YBCO superconductor at 77 K. The assumed parameters for this configuration are

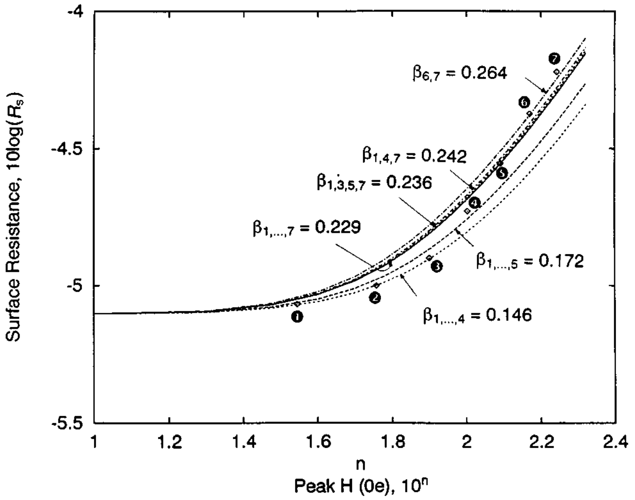


Fig. 1. Surface resistance versus magnetic-field values. Experimental [18] and numerically computed values. The reported values of the nonlinear parameter β have been obtained by a least-squares technique. In particular, the sequence of subscript numbers refers to the corresponding experimental sample values used for the computation.

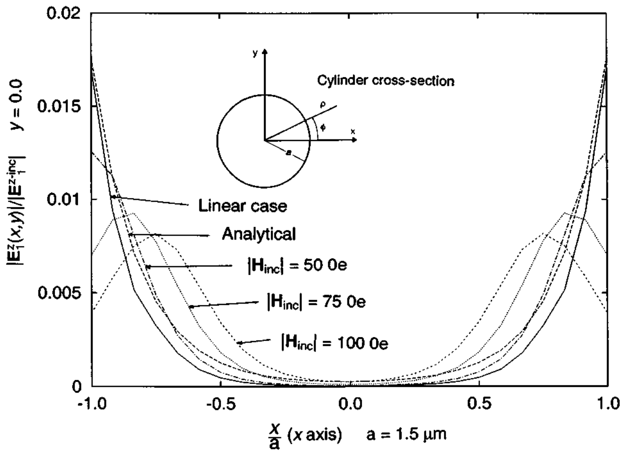
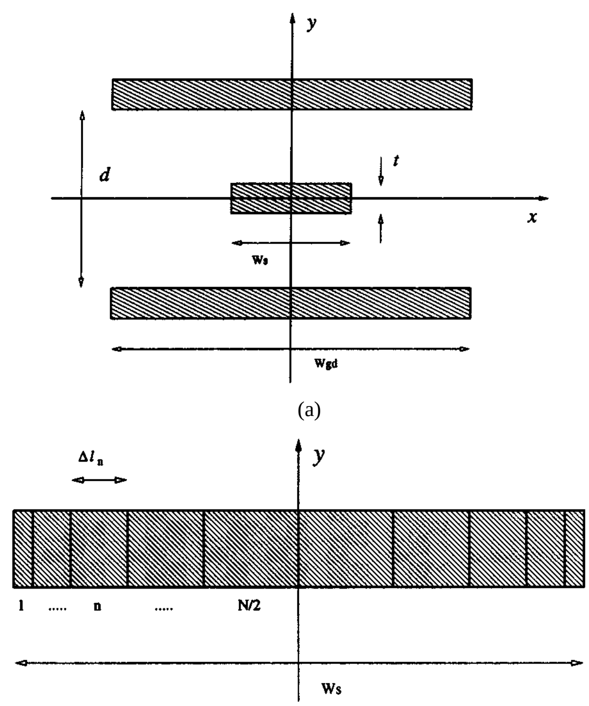


Fig. 2. Scattering by a superconducting circular cylinder ($a = 1.5$ μm; $\sigma_{10} = 5.2624 \cdot 10^5$ S/m, $\mu_r = 1$, $\lambda = 0.25$ μm; 489 subareas). Nonlinear parameter: $\beta = 0.229$. Amplitude values of the total electric field at the fundamental frequency for different values of the magnetic field.



(a)

(b)

Fig. 3. (a) Cross section of the superconducting stripline. (b) Partitioning of the stripline for the numerical computation.

a (radius) $= 1.5$ μm; $\sigma_{10} = 5.2624 \cdot 10^5$ S/m, $\mu_r = 1$, and $\lambda = 0.25$ μm. In order to take into account the effects of the nonlinearity, the nonlinear behavior experimentally obtained in [18] was considered. It should stressed that, very recently, experimental studies concerning more recent HTS films have pointed out a nonlinear behavior much less nonlinear, although still nonlinear [20]–[23], [43]. However, the assumed data can still be used as an example for the evaluation of the applicability of the proposed computational method. In particular, starting from the results in [18, Fig. 8], concerning the film indicated as "Film 1" at 77 K and at a frequency $f = 1.5$ GHz, the value of the nonlinear parameter β was estimated by a least-mean-square identification procedure. We obtained $\beta_{1,\ldots,4} = 0.146$, $\beta_{1,\ldots,5} = 0.172$, $\beta_{1,\ldots,7} = 0.229$, $\beta_{1,3,5,7} = 0.236$, $\beta_{1,4,7} = 0.242$, and $\beta_{1,6,7} = 0.264$, depending on the number of couples of experimental values (magnetic-field strength and surface resistance) used for the computation. The sequence of subscript numbers of the nonlinear parameter corresponds to the sequence of experimental samples considered, as indicated in Fig. 1. The resulting values of the surface resistance, numerically computed on the basis of the above-estimated values of the nonlinear parameter, are also plotted in this figure. For the numerical computation, the cross section was partitioned, according to the role described in [30] for the scattering by dielectric circular cylinders, into 489 cells of area (0.125 μm^2 and the fractional change $\Delta\lambda/\lambda$ was not considered.

Fig. 2 gives the computed values of the amplitude of the total electric field at the fundamental frequency E_1^z, normalized to the amplitude of the incident field for different values of the magnetic-field strength for which the nonlinear behavior was experimentally observed in [18]. For the nonlinear parameter β, we chosen the value $\beta = 0.229$. As can be seen in Fig. 2, the effect of the quadratic nonlinearity on the amplitude of the total electric field was pronounced. In the linear case, the solution was compared with the analytic values obtained by the classic series expansion in terms of Bessel functions.

Moreover, although the nonlinearity was weak, as expected, the iterative process did not converge, as the scatterer was anything but weak. Nevertheless, as shown in [30] and [35], the results at the first iteration step ($k = 1$) can still be considered of interest, as the residual error, defined in [35, eq. (9)] is rather small if the nonlinearity is weak. In the present case, the residual error was less then 0.005 for $k = 1$ and less then 0.08 for $k < 10$. The corresponding error growth is quite slow (e.g., in comparison with the case in [35, Fig. 2(g) and Table 1]) and its value at $k = 1$ is quite small.

In the second example, a more interesting situation, from the point-of-view of the related applications, was considered. In particular, the proposed approach was used for the evaluation

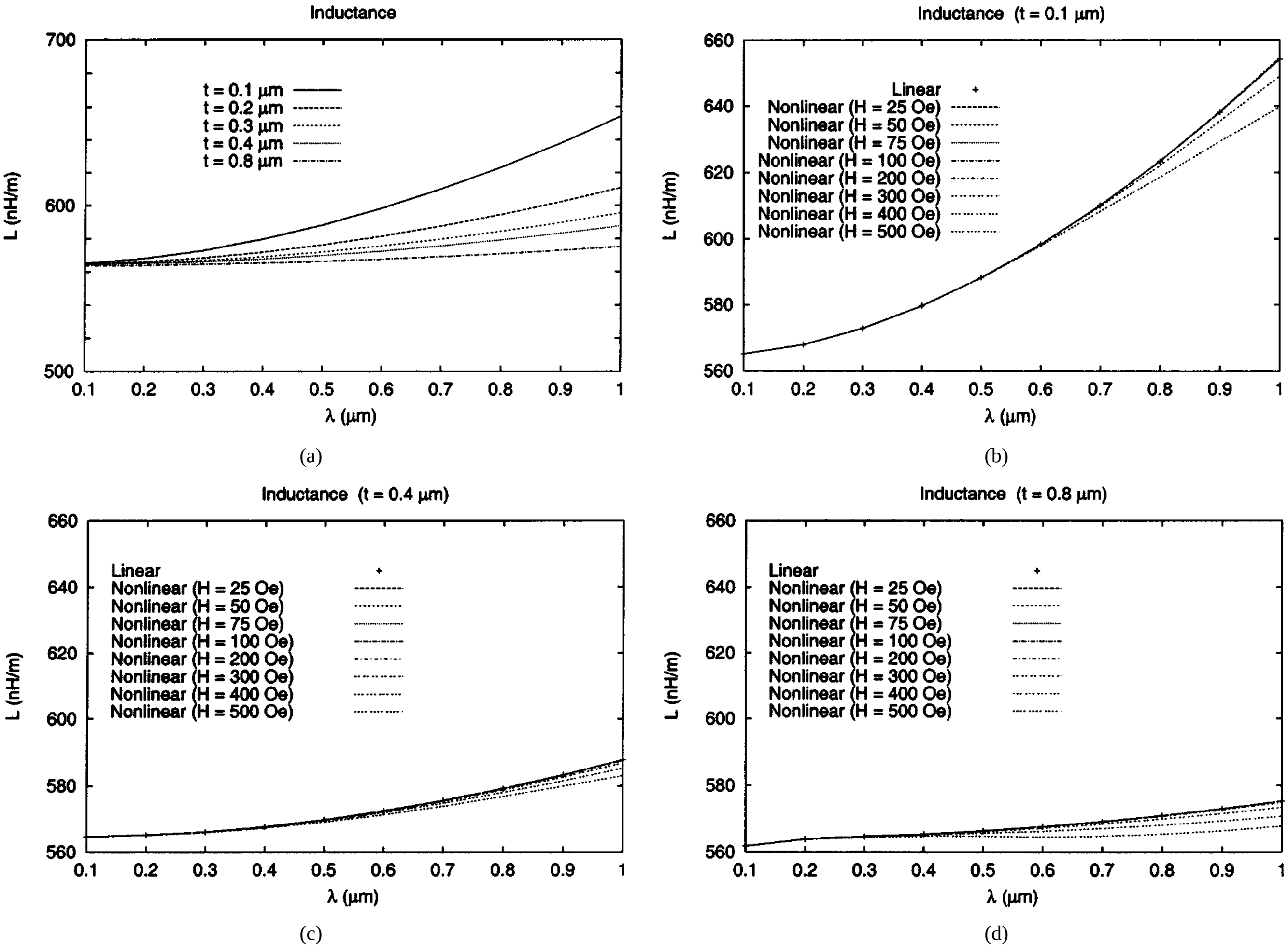


Fig. 4. Inductance per unit length of the stripline versus the penetration depth for different values of the film thickness and for various values of the peak magnetic field. (a) Linear case. (b)–(d) Nonlinear case.

of a propagating structure, i.e., the superconducting stripline shown in Fig. 3(a). The width of the center superconductor was assumed to be equal to $W_s = 150$ μm and the width of the ground planes was $W_{gp} = 8000$ μm. The thickness of the nonlinear superconducting film is indicated by t and the distance between the two return lines is $d = 864$ μm. The central conductor is embedded in vacuum. The same propagating structure was considered in [44], where a method for the calculation of the current distribution, resistance, and inductance matrices for systems of coupled linear superconducting transmission lines having finite rectangular cross sections was proposed.

In order to apply the numerical approach, a nonuniform grid was superimposed to the cross section of the superconducting plates. The propagating structure was analyzed by applying the Weeks method [45], [46] for the evaluation of classical transmission lines, which has been modified in order to take into account the modeling of the superconducting material described in Section II. In particular, the partitioning of the structure was determined so that the grid dimensions were smaller where the current density was larger. Consequently, the discretization patches resulted to be concentrated in the center of the ground planes and at the edges of the center strip. In more details, the smallest grid elements were assumed to be $\Delta l_1 = (\lambda/4)$ inside, with λ being the penetration depth. The side sizes of the other patches were computed according to the following rule:

$$\Delta l_{n+1} = n(\Delta l_1), \qquad n = 2, \ldots, \frac{N}{2} \tag{24}$$

where N indicates the number of discretization cells along the x- or y-directions. The discretization used is also shown in Fig. 3(b).

Fig. 4 gives the computed inductance per unit length as a function of the penetration depth λ. In this case, the reactive effect associated to the fractional change $\Delta\lambda/\lambda$ was neglected and (11) was used with $\varepsilon_{r_{\rm NL}} = -j(\beta H_{\rm rf}^2)/(\omega\varepsilon_0)$. For a consistency check, the computed values in the linear case is shown in Fig. 4(a), which exhibits a good agreement with the results provided in [44, Fig. 5]. Fig. 4(b)–(d) gives the nonlinear values obtained by applying the proposed approach for different values of thickness of the nonlinear superconducting film. It should be noted that the nonlinearity due to the magnetic field essentially affects the kinetic contribution of the inductance of the stripline. Analogous results concerning the behavior of the resistance per unit length are presented in Fig. 5, including a comparison check with the linear values obtained in [44]. Even in this case, there is a good agreement with the plots in [44, Fig. 7].

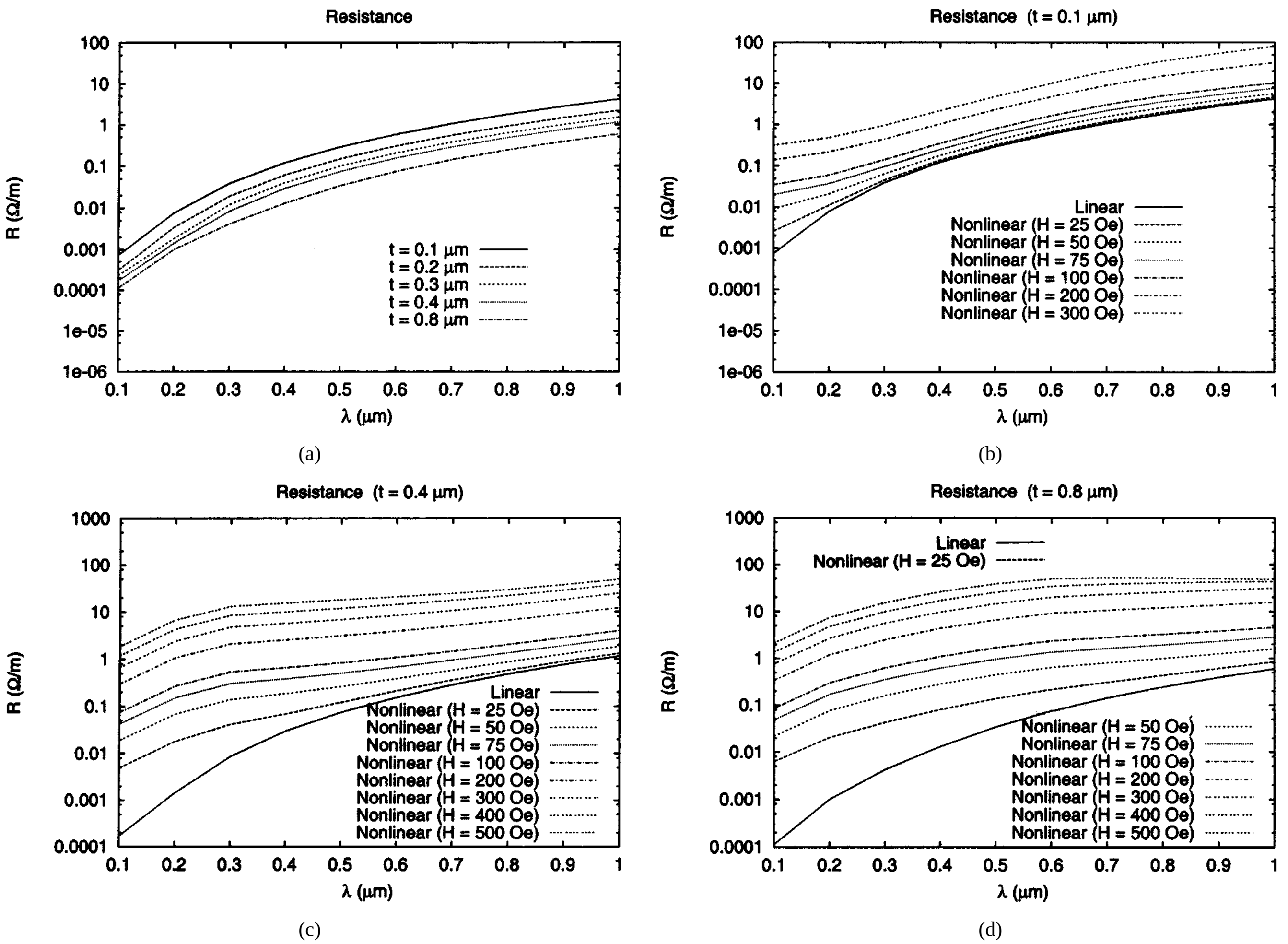


Fig. 5. Resistance per unit length of the stripline versus the penetration depth for different values of the film thickness and for various values of the peak magnetic field. (a) Linear case. (b)–(d) Nonlinear case.

When the fractional change $\Delta\lambda/\lambda$ is taken into account, the model for the nonlinearity is still expressed through (11), but in this case, the nonlinear part of the effective dielectric permittivity is given by $\varepsilon_{r_{\rm NL}} = (2\varsigma H_r f^2)/(\omega\varepsilon_0\mu\lambda_0^2) - j(\beta H_{\rm rf}^2 + \zeta H_{\rm rf}^4)/(\omega\varepsilon_0)$. As expected (see Section II), the contributions of this term to inductance and resistance per unit length turned out to be very limited. Concerning the inductance, the relative change $\Delta L/L$ was found to be less than 2.0×10^{-4} for $t = 0.1$ μm and less than 1.0×10^{-4} for 0.2 μm $\leq t \leq 0.8$ μm and for a peak value of $H_{\rm rf}$ less than 500 Oe. More significant (although still very small) was the effect on the resistance, for which the relative change $\Delta R/R$ was found to be less than 1.0×10^{-2} for 0.1 μm $\leq \lambda_0[= \lambda(H_{\rm rf} = 0)] \leq 0.5$ μm and less than 0.5×10^{-2} for 0.8 μm $\leq \lambda_0 \leq 1$ μm, for $0.1 \leq t \leq 0.8$ μm and for a peak value of $H_{\rm rf}$ less than 500 Oe.

Finally, for comparison purposes, another propagating structure was considered. A microstrip line made of YBCO with a dielectric substrate made of $LaAlO_3(\varepsilon_r = 25)$ is assumed (Fig. 6). The width of the signal and return lines were $W_s = 150$ μm and $W_{gd} = 5000$ μm, respectively. The above structure is the same as considered in [47] and is similar to the one studied in [8]. The analysis is performed for different values of the thickness of the superconducting film in the range from 0.002 to 0.2 μm.

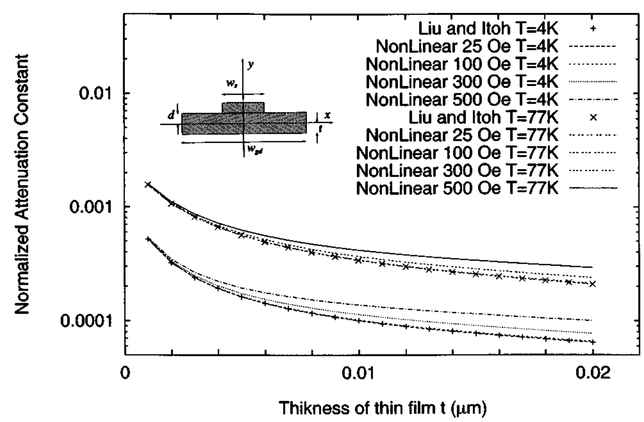


Fig. 6. Normalized attenuation constant of a superconducting microstrip line versus the film thickness for various values of the peak magnetic fields ($d = 0.5$ mm). The results of Liu and Itoh are published in [47].

The results concerning the values of the attenuation constant are provided in Fig. 6 and show a good agreement with those provided in [47, Fig. 6].

IV. Conclusions

In this paper, the interaction between electromagnetic waves at microwave frequencies and superconducting materials has been addressed from a computational point-of-view. Starting from a macroscopic model in which a supercounducting material has been identified as a dielectric material with a negative dielectric permittivity, an integral-equation formulation has been applied, taking into account the nonlinear behavior of superconductors previously experimentally observed by other researchers and justified by them in the light of the rigorous Ginsburg–Landau theory. The integral-equation formulation has been applied in conjunction with an approach based on the distorted-wave Born approximation. Some examples have been provided, concerning both free-space interaction and guided propagation. Although preliminary, these examples seem to demonstrate the possibility of taking into account, from a computational point-of-view, the information and data experimentally obtained, concerning the nonlinear behavior of superconductors versus the magnetic field.

ACKNOWLEDGMENT

The authors would like to express their gratitude to Dr. M. Tsuk, Digital Equipment Corporation, Maynard, MA, for allowing the use of his computer code for the analysis of microstrip configurations and for his contribution in the extension of this code to superconducting materials. Moreover, the authors thank Dr. M. Donelli, University of Genoa, Genoa, Italy, for his support and cooperation.

REFERENCES

[1] H. S. Kwok and D. T. Shaw, *Superconductivity and Its Applications*. Amsterdam, The Netherlands: Elsevier, 1988.

[2] T. Van Duzer and C. W. Turner, *Principle of Superconductive Devices and Circuits*. Amsterdam, The Netherlands: Elsevier, 1981.

[3] J. C. Ritter, M. Nisenoff, G. Price, and S. A. Wolf, "High temperature superconductivity space experiment," *IEEE Trans. Magn.*, vol. 27, pp. 2533–2536, Mar. 1991.

[4] R. S. Withers and R. W. Ralston, "Superconductive analog signal processing," *Proc. IEEE*, vol. 77, pp. 1247–1263, Aug. 1989.

[5] G. C. Liang, "High-temperature superconductor microwave circuits," Ph.D. dissertation, Dept. Elect. Eng. Comput. Sci., Univ. California at Berkeley, Berkeley, CA, 1990.

[6] V. I. Abramov, A. Y. Klimov, A. N. Reznik, and B. B. Tagunov, "Nonlinear dissipation in a miniature superconducting microwave antenna," *Tech. Phys. Lett.*, vol. 21, pp. 203–205, 1995.

[7] L. H. Lee, S. M. Ali, and W. G. Lyons, "Full-wave characterization of high-T_c superconducting transmission lines," *IEEE Trans. Appl. Superconduct.*, vol. 2, pp. 49–57, June 1992.

[8] H. Y. Lee and T. Itoh, "Phenomenological loss equivalence method for planar quasi-TEM transmission lines with a thin normal conductor or superconductor," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 1904–1909, Dec. 1989.

[9] S. El-Ghazaly, R. Hammond, and T. Itoh, "Analysis of superconducting microwave structures: Application to microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 499–508, Mar. 1992.

[10] O. R. Baiocchi, K.-S. Kong, H. Ling, and T. Itoh, "Effects of superconducting losses in pulse propagation on microstrip lines," *IEEE Microwave Guided Wave Lett.*, vol. 1, pp. 2–4, Jan. 1991.

[11] E. B. Ekholm and S. W. McKnight, "Attenuation and dispersion for high-T_c superconducting microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 387–395, Apr. 1990.

[12] J. M. Chwalek *et al.*, "Ultrafast response of superconducting transmission lines," *IEEE Trans. Magn.*, vol. 25, pp. 814–817, Mar. 1989.

[13] D. R. Dykaar *et al.*, "High-frequency characterization of thin-film Y–Ba–Cu oxide superconducting transmission lines," *Appl. Phys. Lett.*, vol. 52, no. 17, pp. 1444–1446, 1988.

[14] W. J. Gallagher *et al.*, "Subpicosecond optoelectronic study of resistive and superconductive transmission lines," *Appl. Phys. Lett.*, vol. 50, no. 6, pp. 350–352, 1987.

[15] M. C. Nuss *et al.*, "Propagation of terahertz bandwidth electrical pulses on YBa2Cu3Oz–delta transmission lines on lanthanum aluminate," *Appl. Phys. Lett.*, vol. 54, no. 22, pp. 2265–2267, 1989.

[16] W. Rauch, E. Gornik, G. Solkner, A. A. Valenzuela, H. Behner, and G. Gieres, "High temperature superconducting coplanar and epitaxially grown microstrip transmission lines studied by a half-wavelength resonator technique," *Appl. Superconduct.*, vol. 2, no. 6, pp. 417–423, June 1994.

[17] K. K. Mei and G. Liang, "Electromagnetics of superconductors," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 1545–1552, Sept. 1991.

[18] D. E. Oates, A. C. Anderson, D. M. Sheen, and S. M. Ali, "Stripline resonator measurements of Z_s versus $H_{\rm rf}$ in $YBa_2Cu_3O_{7-x}$ thin films," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 1522–1529, Sept. 1991.

[19] D. E. Oates, A. C. Anderson, and P. M. Mankiewich, "Measurement of the surface resistance of $YBa_2Cu_3O_{7-x}$ thin films using stripline resonators," *J. Superconduct.*, vol. 3, pp. 251–259, 1991.

[20] D. P. Choudhury, B. A. Willemsen, J. S. Derov, and S. Sridhar, "Nonlinear response of HTSC thin film microwave resonators in an applied DC magnetic field," *IEEE Trans. Appl. Superconduct.*, vol. 7, pp. 1260–1263, June 1997.

[21] G. Muller *et al.*, "On-axis DC sputtered $YBa_2Cu_3O_{7-x}$ films up to 2" in diameter for microwave antenna arrays," *IEEE Trans. Appl. Superconduct.*, vol. 7, pp. 1729–1732, June 1995.

[22] M. Hein *et al.*, "Fundamental limits of the linear microwave power response of epitaxial Y–Ba–Cu–O films," *IEEE Trans. Appl. Superconduct.*, vol. 7, pp. 1236–1239, June 1997.

[23] B. A. Willemsen, J. S. Derov, J. H. Silva, and S. Sridhar, "Nonlinear response of suspended high temperature superconducting thin film microwave resonators," *IEEE Trans. Appl. Superconduct.*, vol. 5, pp. 1753–1755, June 1997.

[24] J.-G. Ma and I. Wolff, "Electrodinamics in high-T_c superconductors," *IEEE Trans. Microwave Theory Tech.*, vol. 44, pp. 537–542, Apr. 1996.

[25] S. Anlage, "Electrodynamics of the electron-doped cuprate superconductors," in *Proc. 6th High-Temperature Superconduct. High-Frequency Fields Symp.*, Naples, Italy, 2000, pp. 24–27.

[26] V. V. Talanov, L. V. Mercaldo, and S. M. Anlage, "Measurement of the absolute penetration depth and surface resistance of superconductors using the variable spacing parallel plate resonator," *IEEE Trans. Appl. Superconduct.*, vol. 9, pp. 2179–2182, June 1999.

[27] L. D. Landau, E. M. Lifshitz, and E. Pitaevski, *Electrodynamics of Continuous Media*. New York: Pergamon, 1984.

[28] K. M. Leung, "Scattering of transverse-electric electromagnetic waves with a finite nonlinear film," *J. Opt. Soc. Amer. B, Opt. Phys.*, vol. 5, pp. 571–574, 1988.

[29] S. Caorsi, A. Massa, and M. Pastorino, "A numerical solution to full-vector electromagnetic scattering by three-dimensional nonlinear bounded dielectrics," *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 428–436, Feb. 1995.

[30] ——, "Approximate solutions to the scattering by nonlinear isotropic dielectric cylinders of circular cross sections under TM illumination," *IEEE Trans. Antennas Propagat.*, vol. 43, pp. 1262–1269, Nov. 1995.

[31] C.-W. Lam, D. M. Sheen, S. M. Ali, and D. E. Oates, "Modeling the nonlinearity of superconducting strip transmission lines," *IEEE Trans. Appl. Superconduct.*, vol. 2, pp. 58–66, June 1992.

[32] D. Mihalanche, D. Mazilu, M. Bortolotti, and C. Sibilla, "Exact solution for nonlinear thin-film guided waves in higher-order nonlinear media," *J. Opt. Soc. Amer. B, Opt. Phys.*, vol. 5, pp. 565–570, 1986.

[33] S. Caorsi, A. Massa, and M. Pastorino, "Electromagnetic scattering by nonlinear dielectric cylinders with non-Kerr-like nonlinearities," *Microwave Opt. Technol. Lett.*, vol. 10, no. 6, pp. 347–350, 1995.

[34] C. T. Tai, *Dyadic Green's Functions in Electromagnetic Theory*. Scranton, PA: International Textbook, 1971.

[35] S. Caorsi, A. Massa, and M. Pastorino, "Iterative numerical computation of the electromagnetic field inside weakly nonlinear infinite dielectric cylinders of arbitrary cross sections using the distorted-wave Born approximation," *IEEE Trans. Microwave Theory Tech.*, vol. 44, pp. 400–412, Mar. 1995.

[36] M. A. Hasan and P. L. E. Uslenghi, "Electromagnetic scattering from nonlinear anisotropic cylinders—Part I: Fundamental frequency," *IEEE Trans. Antennas Propagat.*, vol. 38, pp. 523–533, Apr. 1990.

[37] F. Gao, Z. Zhou, M. Feng, M. Scharen, A. Cardona, and R. Forse, "Hysteresis effect in microwave power transmission of high temperature superconducting coplanar transmission lines," *Appl. Phys. Lett.*, vol. 67, no. 15, pp. 2229–2231, 1995.

[38] J. H. Richmond, "Scattering by a dielectric cylinder of arbitrary cross-section shape," *IEEE Trans. Antennas Propagat.*, vol. AP-13, pp. 334–341, May 1965.

[39] A. F. Peterson, "Analysis of heterogeneous electromagnetic scatterers: Research progress of the past decade," *Proc. IEEE*, vol. 79, pp. 1431–1453, Oct. 1991.

[40] E. H. Newman and M. Kragalott, "Moment method analysis of the electric shielding factor of a conducting TM shield at ELF," *IEEE Trans. Electromagn. Compat.*, vol. 37, pp. 400–408, Aug. 1995.

[41] S. Caorsi, A. Massa, and M. Pastorino, "Optimization procedure based on a statistical cooling method applied to scattering by bounded nonlinear objects," *Radio Sci.*, vol. 31, pp. 437–450, 1996.

[42] R. J. Wombell and R. D. Murch, "The reconstruction of dielectric objects from scattered field data using the distorted-wave Born approximation," *J. Electromag. Waves Applicat.*, vol. 7, pp. 687–702, 1983.

[43] H. Xin *et al.*, "Comparison of power dependence of microwave surface resistance of unpatterned and patterned YBCO thin film," *IEEE Trans. Microwave Theory Tech.*, vol. 48, pp. 1221–1226, July 2000.

[44] D. M. Sheen, S. M. Ali, D. E. Oates, R. S. Withers, and J. A. Kong, "Current distribution, resistance and inductance for superconducting strip transmission lines," *IEEE Trans. Appl. Superconduct.*, vol. 1, pp. 108–115, June 1991.

[45] W. T. Weeks, L. L. Wu, M. F. McAllister, and A. Singh, "Resistive and inductive skin effect in rectangular conductors," *IBM J. Res. Develop.*, vol. 23, no. 6, pp. 652–660, 1979.

[46] M. J. Tsuk and J. A. Kong, "A hybrid method for the calculation of the resistance and inductance of transmission lines with arbitrary cross sections," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 1338–1347, Aug. 1991.

[47] Y. Liu and T. Itoh, "Characterization of power-dependent high-T_c superconducting microstrip line by modified spectral domain method," *Radio Sci.*, vol. 28, pp. 913–918, 1993.

Salvatore Caorsi (M'98) received the Laurea degree in electronic engineering from the University of Genoa, Genoa, Italy, in 1973.

Since 1994, he has been a Full Professor of Electromagnetic Compatibility in the Department of Electronics, University of Pavia, Pavia, Italy. He also teaches the "Antennas" course at the University of Genoa. His primary activities are vision and remote sensing, biology, and medicine. He is particularly involved with research project concerning human hazards to electromagnetic exposure, numerical methods for solving electromagnetic problems, wave interaction in the presence of nonlinear media, inverse scattering and microwave imaging, and electromagnetic compatibility.

Prof. Caorsi is a member of the Elettrotecnica ed Elettronica Italiana (AEI), the European Bioelectromagnetic Association (EBEA), and the European Society for Hyperthermic Oncology (ESHO). He is also the past president and founding member of the Inter-University Research Center for the Interactions Between Electromagnetic Fields and Biological Systems (ICEmB).

Andrea Massa received the Laurea degree in electronic engineering and the Ph.D. degree in electronics and computer science from the University of Genoa, Genoa, Italy, in 1992 and 1996, respectively.

From 1997 to 1999, he was an Assistant Professor of electromagnetic fields in the Department of Biophysical and Electronic Engineering, University of Genoa, where he taught the "Electromagnetic Fields 1" course. He is currently an Assistant Professor at the University of Trento, Trento, Italy, where he teaches "Electromagnetic Fields" and "Electromagnetic Diagnostic Techniques" courses. Since 1993, his research has been principally involved with electromagnetic direct and inverse scattering, optimization techniques for microwave imaging, wave propagation in the presence of nonlinear media, and applications of electromagnetic fields to telecommunications, medicine, and biology.

Dr. Massa is a member of the Inter-University Research Center for Interaction Between Electromagnetic Fields and Biological Systems (ICEmB).

Matteo Pastorino (M'90–SM'96) received the Laurea degree in electronic engineering and the Ph.D. degree in electronics and computer science from the University of Genoa, Genoa, Italy, in 1987 and 1992, respectively.

He is currently an Associate Professor of electromagnetic fields in the Department of Biophysical and Electronic Engineering, University of Genoa, where he is also in charge of the Applied Electromagnetics Group and teaches "Electromagnetic Fields" and "Remote Sensing and Electromagnetic Diagnostics" courses. His main research interests are in the field of electromagnetic direct and inverse scattering, microwave imaging, wave propagation in the presence of nonlinear media, and analytical and numerical methods in electromagnetism.

Prof. Pastorino is a member of the Associazione Elettrotecnica ed Elettronica Italiana (AEI) and the European Bioelectromagnetic Association (EBEA). He is member of the IEEE Instrumentation and Measurement Technical Committee on Imaging Systems.